

### Calculus Warm Up #4-5

The rate at which people enter an auditorium for a rock concert is modeled by the function  $R$ ,

$$R(t) = 1380t^2 - 675t^3, \text{ for } 0 \leq t \leq 2 \text{ hours.}$$

No one is in the auditorium at time  $t = 0$ . Doors close and concert begins at time  $t = 2$ .

The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the the person enters the auditorium and ending when the concert begins. The function  $w$  models the total wait time for all the people who enter before time  $t$ .

$$w'(t) = (2 - t)R(t)$$

Find:  $w(2) - w(1)$ , the total wait time for those who enter after time  $t = 1$ .

Staple and turn in:

Week 4 Classwork

Warm up on top

Purple FR 2002, # 1-3

Blue FR -B 2002, # 6

Today: 10.6 The Ratio Test

Group Quiz on 10.1 - 10.4

Check answers on MC practice

Start HW

## 10.6 The Ratio Test

A test for absolute convergence for  $\sum a_n$

1) Absolute convergence if  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$

2) Diverges if  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| > 1$

3) Inconclusive if  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$

Ex:  $\sum_{n=0}^{\infty} \frac{3^n}{5^n + 1}$

Ratio Test

$$\begin{aligned} & \lim_{n \rightarrow \infty} \left( \left( \frac{3^{n+1}}{5^{n+1} + 1} \right) \div \left( \frac{3^n}{5^n + 1} \right) \right) \\ &= \lim_{n \rightarrow \infty} \left( \frac{3^{n+1}}{5^{n+1} + 1} \cdot \frac{5^n + 1}{3^n} \right) \\ &= \lim_{n \rightarrow \infty} \frac{3(5^n + 1)}{5^{n+1} + 1} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} \\ &= 3 \lim_{n \rightarrow \infty} \left( \frac{1 + \frac{1}{5^n}}{5 + \frac{1}{5^n}} \right) \\ &= \frac{3}{5} < 1, \therefore \text{Series} \\ & \quad \text{Converges.} \end{aligned}$$

You try:

$$\sum_{n=0}^{\infty} \frac{n^2 \cdot 2^{n+1}}{3^n}$$

Ratio Test

$$\begin{aligned} & \lim_{n \rightarrow \infty} \left( \frac{(n+1)^2 2^{n+2}}{3^{n+1}} \cdot \frac{3^n}{n^2 \cdot 2^{n+1}} \right) \\ &= \lim_{n \rightarrow \infty} \frac{2(n+1)^2}{3n^2} \\ &= \frac{2}{3} < 1 \\ &\therefore \sum_{n=0}^{\infty} \frac{n^2 \cdot 2^{n+1}}{3^n} \\ & \quad \text{converges} \end{aligned}$$

last one:

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$$

Ratio Test  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$

$$\lim_{n \rightarrow \infty} \left( \frac{\sqrt{n+1}}{(n+2)} \cdot \frac{n+1}{\sqrt{n}} \right)$$

$$a_{n+1} = \frac{\sqrt{n+1}}{(n+1)+1}$$

$$= \lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{n}} \cdot \lim_{n \rightarrow \infty} \frac{n+1}{n+2} \cdot \frac{1}{n}$$

$$= \lim_{n \rightarrow \infty} \sqrt{1 + \frac{1}{n}} \cdot \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{1 + \frac{2}{n}}$$

$$= 1 \cdot 1 = 1$$

Ratio Test is inconclusive.

## Classwork: BC MC - A Practice test

\*Things we still need to learn:

#7, 10, 17, 26 (#13: derivatives of parametric equations. Try it!)

check answers:

1. D	6. B	11. B	16. E	21. E	26. C
2. C	7. B	12. D	17. D	22. A	27. E
3. C	8. D	13. B	18. B	23. B	28. A
4. E	9. A	14. C	19. D	24. D	
5. D	10. D	15. E	20. E	25. E	

HW: p. 604, # 1 - 19 odd

Simplifying  $\frac{n!}{(n+1)!}$

$$= \frac{n(n-1)(n-2)(n-3)\dots}{(n+1)(n+1-1)(n+1-2)(n+1-3)\dots}$$

$$= \frac{\cancel{n}(\cancel{n-1})(\cancel{n-2})(\cancel{n-3})\dots}{(n+1)(\cancel{n})(\cancel{n-1})(\cancel{n-2})\dots}$$

$$= \frac{1}{n+1} \quad \text{"}$$